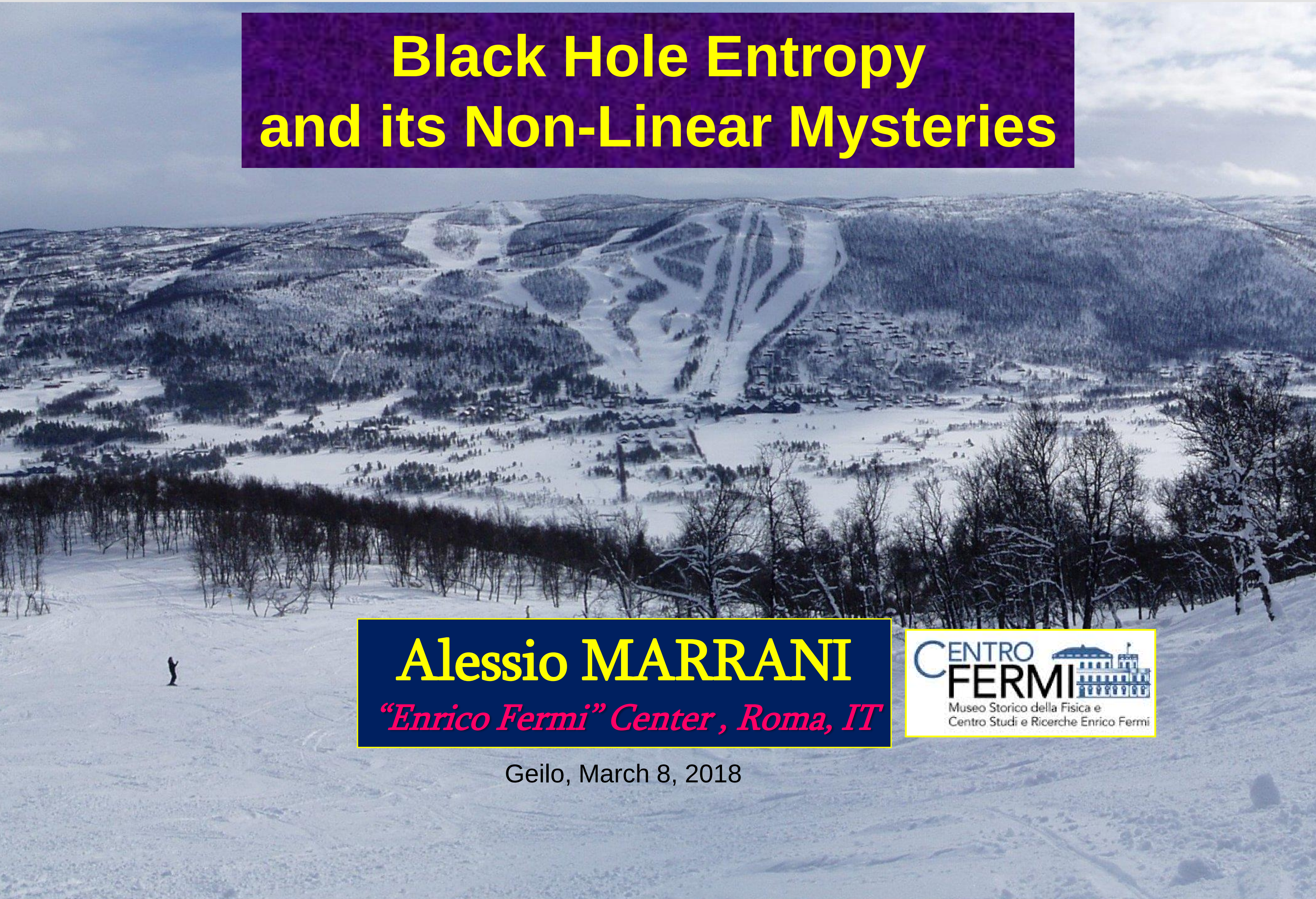

Black Hole Entropy and its Non-Linear Mysteries

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Summary

Maxwell-Einstein-Scalar Gravity Theories

Symmetric Scalar Manifolds :
Application to (**Super)Gravity** and **Extremal Black Holes**

Attractor Mechanism

The matrix $\mathcal{M}$ and **Freudenthal Duality**

Groups "of type E_7"

Maxwell-Einstein-Scalar Theories

$$\mathcal{L} = -\frac{R}{2} + \frac{1}{2} g_{ij}(\varphi)\,\partial_\mu \varphi^i \partial^\mu \varphi^j + \frac{1}{4} I_{\Lambda\Sigma}(\varphi)\, F^\Lambda_{\mu\nu} F^{\Sigma|\mu\nu} + \frac{1}{8\sqrt{-G}} R_{\Lambda\Sigma}(\varphi)\, \epsilon^{\mu\nu\rho\sigma} F^\Lambda_{\mu\nu} F^\Sigma_{\rho\sigma}$$

D=4 Maxwell-Einstein-scalar system (with no potential)

[may be the bosonic sector of **D=4** (ungauged) **sugra**]

$$H := \left(F^\Lambda, G_\Lambda\right)^T;$$

$$^*G_{\Lambda|\mu\nu} := 2\frac{\delta \mathcal{L}}{\delta F^{\Lambda|\mu\nu}}.$$

Abelian 2-form field strengths

static, spherically symmetric, asymptotically flat, **extremal BH**

$$ds^2 = -e^{2U(\tau)} dt^2 + e^{-2U(\tau)} \left[\frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2}\left(d\theta^2 + \sin\theta d\psi^2\right) \right]$$

$$\tau := -1/r$$

$$\mathcal{Q} \ : = \int_{S^2_\infty} H = \left(p^\Lambda, q_\Lambda\right)^T;$$

$$p^\Lambda \ : = \frac{1}{4\pi} \int_{S^2_\infty} F^\Lambda, \ q_\Lambda = \frac{1}{4\pi} \int_{S^2_\infty} G_\Lambda.$$

dyonic vector of electric & magnetic fluxes (**BH charges**)

$$S_{D=1} = \int [(U')^2 + g_{ij}\varphi'^i\varphi'^j + e^{2U}V_{BH}(\varphi(\tau), \mathcal{Q})]d\tau$$

$$' \equiv \frac{d}{d\tau}$$

reduction D=4 → D=1 : **effective** 1-dimensional (radial) Lagrangian

$$V_{BH}(\varphi, \mathcal{Q}) := -\frac{1}{2}\mathcal{Q}^T\mathcal{M}(\varphi)\mathcal{Q},$$

BH effective potential

Ferrara, Gibbons, Kallosh

eoms

$$\begin{cases} \frac{d^2U}{d\tau^2} = e^{2U}V_{BH}; \\ \frac{d^2\varphi^i}{d\tau^2} = g^{ij}e^{2U}\frac{\partial V_{BH}}{\partial\varphi^j}. \end{cases}$$

Attractor Mechanism : $\partial_\varphi V_{BH} = 0 \Leftrightarrow lim_{\tau\to-\infty}\varphi^a(\tau) = \varphi^a_H(\mathcal{Q})$

conformally flat geometry $AdS_2 \times S^2$ near the horizon $ds^2_{\text{B-R}} = \frac{r^2}{M^2_{\text{B-R}}}dt^2 - \frac{M^2_{\text{B-R}}}{r^2}(dr^2 + r^2 d\Omega)$

near the horizon, the scalar fields are **stabilized** purely in terms of charges

$$S = \frac{A_H}{4} = \pi\, V_{BH}|_{\partial_\varphi V_{BH}=0} = -\frac{\pi}{2}\mathcal{Q}^T\mathcal{M}_H\mathcal{Q}$$

Bekenstein-Hawking entropy-area formula for extremal dyonic BH

Symmetric Scalar Manifolds

Let's specialize, for a moment, the discussion to theories with scalar manifolds which are **symmetric cosets G/H**

[**N>2** : general, **N=2** : particular, **N=1** : special cases]

H = isotropy group = *linearly* realized; scalar fields sit in an **H**-repr.

In general : **G** = (global) electric-magnetic duality group [in string theory : **U-duality**]

G is an *on-shell* symmetry of the Lagrangian

The 2-form field strengths (F,G) vector and the BH e.m. charges sit in a **G**-repr. **R** which is **symplectic** :

$$\exists ! \mathbb{C}_{[MN]} \equiv \mathbf{1} \in \mathbf{R} \times_a \mathbf{R};$$

$$\langle \mathcal{Q}_1, \mathcal{Q}_2 \rangle \equiv \mathcal{Q}_1^M \mathcal{Q}_2^N \mathbb{C}_{MN} = -\langle \mathcal{Q}_2, \mathcal{Q}_1 \rangle$$

symplectic product

$$\mathbb{C} = \begin{pmatrix} \mathbf{0}_n & \mathbb{I}_n \\ -\mathbb{I}_n & \mathbf{0}_n \end{pmatrix}$$

$$\begin{array}{rcl} G & \subset & Sp(2n, \mathbb{R}); \\ \mathbf{R} & = & \mathbf{2n} \end{array}$$

Gaillard-Zumino embedding (generally maximal, but not symmetric)

Dynkin, Gaillard-Zumino

❖ **symmetric** vector mults' scalar mfds of **N=2, D=4 sugra [PVS]**

	$\frac{G_V}{H_V}$	r	$dim_{\mathbb{C}} \equiv n_V$
quadratic sequence $n \in \mathbb{N}$	$\frac{SU(1,n)}{U(1)\otimes SU(n)}$	1	n
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,n)}{SO(2)\otimes SO(n)}$	$2\ (n=1)$ $3\ (n \geqslant 2)$	$n+1$
$J_3^{\mathbb{O}}$	$\frac{E_{7(-25)}}{E_{6(-78)}\otimes U(1)}$	3	27
$J_3^{\mathbb{H}}$	$\frac{SO^*(12)}{U(6)}$	3	15
$J_3^{\mathbb{C}}$	$\frac{SU(3,3)}{S(U(3)\otimes U(3))} = \frac{SU(3,3)}{SU(3)\otimes SU(3)\otimes U(1)}$	3	9
$J_3^{\mathbb{R}}$	$\frac{Sp(6,\mathbb{R})}{U(3)}$	3	6

another isolated model ($F = T^3$ model) : $\frac{G_V}{H_V} = \frac{SL(2,\mathbb{R})}{U(1)}$, $r = 1$, $\dim_{\mathbb{C}} = 1$

let's reconsider the starting **Maxwell-Einstein-scalar Lagrangian density**

$$\mathcal{L} = -\frac{R}{2} + \frac{1}{2} g_{ij}(\varphi)\,\partial_\mu \varphi^i \partial^\mu \varphi^j + \frac{1}{4} I_{\Lambda\Sigma}(\varphi)\, F^\Lambda_{\mu\nu} F^{\Sigma|\mu\nu} + \frac{1}{8\sqrt{-G}} R_{\Lambda\Sigma}(\varphi)\, \epsilon^{\mu\nu\rho\sigma} F^\Lambda_{\mu\nu} F^\Sigma_{\rho\sigma}$$

…and introduce the following real 2n x 2n matrix :

$$\mathcal{M} = \begin{pmatrix} \mathbb{I} & -R \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & I^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{I} & 0 \\ -R & \mathbb{I} \end{pmatrix} = \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix}$$

$$\mathcal{M} = \mathcal{M}(R, I) = \mathcal{M}(\mathrm{Re}(\mathcal{N}), \mathrm{Im}(\mathcal{N})).$$

$$\mathcal{M}^T = \mathcal{M} \qquad \mathcal{M}\mathbb{C}\mathcal{M} = \mathbb{C}$$

…by virtue of this matrix, one can introduce a (scalar-dependent) **anti-involution in *any* Maxwell-Einstein-scalar gravity theory** with **symplectic structure**, named (**scalar-dependent**) **Freudenthal duality (F-duality)** :

$$\mathfrak{F} := -\mathbb{C}\mathcal{M}(\varphi)$$

Ferrara,AM,Yeranyan; Borsten,Duff, Ferrara,AM

$$\mathfrak{F}^2 = \mathbb{C}\mathcal{M}(\varphi)\,\mathbb{C}\mathcal{M}(\varphi) = \mathbb{C}^2 = -Id$$

By recalling $V_{BH}(\varphi, \mathcal{Q}) := -\frac{1}{2}\mathcal{Q}^T\mathcal{M}(\varphi)\,\mathcal{Q},$

Freudenthal duality can be related to the **effective BH potential** :

$$\mathfrak{F} : \mathcal{Q} \rightarrow \mathfrak{F}(\mathcal{Q}) := \mathbb{C}\frac{\partial V_{BH}}{\partial \mathcal{Q}}.$$

All this enjoys a remarkable physical interpretation when evaluated **at the horizon** :

Attractor Mechanism $\partial_\varphi V_{BH} = 0 \Leftrightarrow lim_{\tau \to -\infty} \varphi^a(\tau) = \varphi^a_H(\mathcal{Q})$

Bekenstein-Hawking entropy $S = \frac{A_H}{4} = \pi V_{BH}|_{\partial_\varphi V_{BH}=0} = -\frac{\pi}{2}\mathcal{Q}^T \mathcal{M}_H \mathcal{Q}$

…by evaluating the matrix M at the horizon $\lim_{\tau \to -\infty} \mathcal{M}(\varphi(\tau)) = \mathcal{M}_H(\mathcal{Q})$

one can define the **horizon Freudenthal duality** as:

$$\lim_{\tau \to -\infty} \mathfrak{F}(\mathcal{Q}) =: \mathfrak{F}_H(\mathcal{Q}) = -\mathbb{C}\mathcal{M}_H \mathcal{Q} = \frac{1}{\pi}\mathbb{C}\frac{\partial S_{BH}}{\partial \mathcal{Q}} =: \tilde{\mathcal{Q}},$$

$$\mathfrak{F}^2_H(\mathcal{Q}) = \mathfrak{F}_H(\tilde{\mathcal{Q}}) = -\mathcal{Q}$$

non-linear (scalar-independent) anti-involutive map on Q (hom of degree one)

Bek.-Haw. entropy is **invariant** under its **non-linear symplectic gradient** (defined by **F-duality**) :

$$S(\mathcal{Q}) = S(\mathfrak{F}_H(\mathcal{Q})) = S\left(\frac{1}{\pi}\mathbb{C}\frac{\partial S}{\partial \mathcal{Q}}\right) = S(\tilde{\mathcal{Q}})$$

This can be extended to include *at least* **all quantum corrections** with **homogeneity 2** or **0** in the BH charges Q

Ferrara, AM, Yeranyan
(and late Raymond Stora)

Lie groups "of type E_7" : (G,**R**)

Brown (1967);
Garibaldi; Krutelevich;
Borsten,Duff *et al.*
Ferrara,Kallosh,AM;
AM,Orazi,Riccioni

- the (ir)repr. **R** is **symplectic** :

$$\exists ! \mathbb{C}_{[MN]} \equiv \mathbf{1} \in \mathbf{R} \times_a \mathbf{R};$$

$$\langle Q_1, Q_2 \rangle \equiv Q_1^M Q_2^N \mathbb{C}_{MN} = - \langle Q_2, Q_1 \rangle ;$$

symplectic product

- the (ir)repr. admits a unique completely symmetric **invariant rank-4** tensor

$$\exists ! \ K_{MNPQ} = K_{(MNPQ)} \equiv \mathbf{1} \in [\mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}]_s$$

(K-tensor)

G-invariant quartic polynomial

$$I_4 := K_{MNPQ} Q^M Q^N Q^P Q^Q =: \epsilon \, |I_4| \, ,$$

$$S_{BH} = \pi \sqrt{|I_4|}$$

- defining a triple map in **R** as

$$T \colon \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}$$

$$\langle T(Q_1, Q_2, Q_3), Q_4 \rangle \equiv K_{MNPQ} Q_1^M Q_2^N Q_3^P Q_4^Q$$

it holds

$$\langle T(Q_1, Q_1, Q_2), T(Q_2, Q_2, Q_2) \rangle = \langle Q_1, Q_2 \rangle \, K_{MNPQ} Q_1^M Q_2^N Q_2^P Q_2^Q$$

this third property makes a **group of type E_7** amenable to a description in terms of **Freudenthal triple systems**

All electric-magnetic **(U-)duality** groups of **D=4** sugras with **symmetric** scalar manifolds and *at least* **8** supersymmetries are **"of type E_7"**

$N = 2$

G	R
$U(1,n)$	$(\mathbf{1+n})$
$SL(2,\mathbb{R}) \times SO(2,n)$	$(\mathbf{2,2+n})$
$SL(2,\mathbb{R})$	$\mathbf{4}$
$Sp(6,\mathbb{R})$	$\mathbf{14'}$
$SU(3,3)$	$\mathbf{20}$
$SO^*(12)$	$\mathbf{32}$
$E_{7(-25)}$	$\mathbf{56}$

N	G	R
3	$U(3,n)$	$(\mathbf{3+n})$
4	$SL(2,\mathbb{R}) \times SO(6,n)$	$(\mathbf{2,6+n})$
5	$SU(1,5)$	$\mathbf{20}$
6	$SO^*(12)$	$\mathbf{32}$
8	$E_{7(7)}$	$\mathbf{56}$

"degenerate" groups "of type E_7"

$$I_4(p,q) = (I_2(p,q))^2$$

$$S_{BH} = \pi\sqrt{|I_4(p,q)|} = \pi\,|I_2(p,q)|\,.$$

In sugras with electric-magnetic duality group **“of type E_7”**, the **G**-invariant **K-tensor** determining the extremal BH Bekenstein-Hawking entropy

$$S_{BH} = \pi\sqrt{|I_4|}$$

$$I_4 := K_{MNPQ}\mathcal{Q}^M\mathcal{Q}^N\mathcal{Q}^P\mathcal{Q}^Q =: \epsilon\,|I_4|\,,$$

can generally be expressed as adjoint-trace of the product of **G**-generators (dim **R** = 2n, and dim **Adj** = d)

$$K_{MNPQ} = -\frac{n\,(2n+1)}{6d}\left[t^{\alpha}_{MN}t_{\alpha|PQ} - \frac{d}{n(2n+1)}\mathbb{C}_{M(P}\mathbb{C}_{Q)N}\right]$$

The **horizon Freudenthal duality** can be expressed in terms of the **K-tensor**

$$\mathfrak{F}_H(\mathcal{Q})_M = \tilde{\mathcal{Q}}_M = \frac{\partial\sqrt{|I_4(\mathcal{Q})|}}{\partial\mathcal{Q}^M} = \epsilon\frac{2}{\sqrt{|I_4(\mathcal{Q})|}}K_{MNPQ}\mathcal{Q}^N\mathcal{Q}^P\mathcal{Q}^Q$$

Borsten,Dahanayake,Duff,Rubens

the **invariance** of the BH entropy under **horizon Freudenthal duality** reads as

$$I_4(\mathcal{Q}) = I_4(\mathbb{C}\tilde{\mathcal{Q}}) = I_4\left(\mathbb{C}\frac{\partial\sqrt{|I_4(\mathcal{Q})|}}{\partial\mathcal{Q}}\right)$$

Thank You!