Abstracts GAPDE 2017

Dmitri Alekseevsky. **Decomposable (4,7) solutions of 11D supergravity.**

We construct solutions of 11D supergravity in the case when the Lorentz 11D manifold is a Riemannian product (M',g')x(M,g) of a Lorentzian 4-manifold (M',g') and a compact Riemannian 7-manifold (M,g) and the flux 4-form $F'+F$ is a direct sum of 4-forms on M' and M. We show that then supergravity equations reduces to Einstein equation with negative Einstein constant for metric g' and some generalized Einstein equation for the Riemann manifold (M,g) with a special 3-form $\boldsymbol{\varphi}$ which satisfies the following Maxwell equation $d\boldsymbol{\varphi} = \lambda\star\boldsymbol{\varphi}$.

We construct some non homogeneous solutions of these equations and then concentrate on the case when the internal space $(M,g,\boldsymbol{\varphi})$ is a homogeneous 7-manifold. In particular, we show that any weakly parallel but not parallel homogeneous G_2 manifold gives a solution of the supergravity equations.

Based on a joint work with Ioannis Chrysikos and Arman Taghavi-Chabert.

Pavel Bibikov. **On second order linear differential equations with rational coefficients.**

In the present work we study linear ordinary differential equations of second order with rational coefficients. Such equations are very important in complex analysis (for example they include the so-called Fuchs equations, which appear in 21 Hilbert's problem) and in the theory of special functions (Bessel function, Gauss hypergeometric function, etc). We compute the symmetry group of this class of equations, and it appears that this group includes non-rational (and even non-algebraic) transformations. Also, the field of differential invariants is described and the effective equivalence criterion is obtained. Finally, we present some examples.

Francesco Cattafi. **Almost Gamma-structures and Pfaffian groupoids.**

It is known that the classical theory of (integrable) G-structures can be equivalently described introducing the concepts of **Γ**-atlases and **Γ**-structures, for a transitive pseudogroup **Γ**. In this talk I will review these objects and present some new results regarding the further generalisation of this theory when **Γ** is a generic (nontransitive) Lie pseudogroup. In particular, in order to define a suitable notion of "almost **Γ**-structure", the natural working framework will be that of Pfaffian groupoids. I will conclude by checking that everything agrees with the known results in the particular case of G-structures, and by explaining how to use this new approach for a further understanding of (obstructions to) integrability. This is a joint work with Marius Crainic.

Evgeny Ferapontov. **Dispersionless integrable hierarchies and *GL*(2,R) geometry.**

In this talk I will discuss a natural class of $GL(2,\mathbf{R})$ structures supported on solutions of integrable hierarchies of the dispersionless Kadomtsev-Petviashvili (dKP) type. Such structures are necessarily involutive, which generalizes the Einstein-Weyl property. Based on joint results with B.Kruglikov.

Nabil Kahouadji. **Isometric Immersions of Pseudo-Spherical Surfaces via Partial Differential Equations.**

There is a remarkable connection between the solutions of the sine-Gordon equation and pseudo-spherical surfaces (PSS), in the sense that every generic solution of this equation can be shown to give rise to a PSS. Furthermore, the sine-Gordon equation has the property that the way in which the pseudo-spherical surfaces corresponding to its solutions are realized geometrically in 3d space is given in closed form through some remarkable explicit formulas. The sine-Gordon equation is but one member of a very large class of differential equations whose solutions likewise define PSS. These were defined and classified by Chern, Tenenblat and others, including many already known examples of integrable PDEs. This raises the question of whether the other equations enjoy the same remarkable property as the sine-Gordon equation when it comes to the realization of the corresponding surfaces in 3d space. We will see that the answer is no, and that the sine-Gordon equation is therefore quite unique in this regard amongst all integrable equations.

Igor Khavkine. **A synthetic approach to the formal theory of PDEs.**
We give an abstract formulation of the formal theory PDEs in synthetic differential geometry (SDG), one that would seamlessly generalize the traditional theory to a range of enhanced contexts, such as super- or higher (stacky) geometry. SDG greatly enriches the notion of a smooth space, admitting both infinite dimensional manifolds and manifolds with infinitesimal directions, in the same way as ordinary manifolds. This freedom allows us to give a precise definition of (a parametrized family of) formal solutions and to directly define the category of PDEs to consist of morphisms that preserve formal solutions, without invoking the Cartan distribution. Within this framework, we prove that the prolongation of a PDE coincides with its universal family of formal solutions and, inspired by the work of Marvan, that our PDE category is equivalent to the (Eilenberg-Moore) category of coalgebras over the jet functor comonad. This is joint work with Urs Schreiber arXiv:1701.06238.

Boris Khesin. **Hamiltonian dynamics of vortex membranes.**
We show that an approximation of the hydrodynamical Euler equation describes the binormal flow on vortex membranes in any dimension. This generalizes the classical binormal, or vortex filament, equation in 3D. We present a Hamiltonian framework for dynamics of higher-dimensional vortex filaments and vortex sheets as singular 2-forms (Green currents) with support of codimensions 2 and 1, respectively. We also discuss the Hamiltonian properties of the Madelung transform relating this dynamics to the NLS-type equations.

Andrey Krutov. **On integrability of new fifth order N=1 supersymmetric evolution equation.**
We discus integrability properties of recently discovered fifth order N=1 supersymmetric evolution equation. Namely, we construct its **sl**(2;1)-valued zero-curvature representation (ZCR) with non-removable parameter via cohomological techniques. We show that this ZCR yields infinity many non-trivial conservation laws.

Radoslaw Kycia. **Analysis of totally geodesic vacuum metrics resulting from the Weyl tensor decomposition**.
Preliminary results on analysis of totally geodesic solutions resulted from the vacuum Einstein's equation with cosmological constant will be provided. The class of metric was described in: Valentin Lychagin, Valeriy Yumaguzhin, Differential invariants and exact solutions of the Einstein equations, Anal.Math.Phys. DOI 10.1007/s13324-016-0130-z.

Arnfinn Laudal. **The noncom phase functor and dynamics of representations.**
I will overview Chapter 4 of the newly published book Eivind Eriksen, Olav Arnfinn Laudal, Arvid Siqveland, Noncommutative Deformation Theory, Chapman&Hall (2017).

Alex Malakhov. **On second order differential equations and Cremona group.**
We consider the class of differential equations $y''=F(x,y)$ with rational function F study and its symmetry group. This group is a subgroup in the Cremona group of birational automorphisms of $\mathbf{C}^2$, which makes it possible to apply for their study methods of differential invariants and geometric theory of differential equations. Also, using algebraic methods in the theory of differential equations we obtain a global classification for such equations instead of local classifications for such problems provided by Lie, Tresse and others.

Irina Markina. **On Cauchy-Szegeo kernel for quaternionic Siegel upper half space.**
In the talk we introduce a quaternionic analogue of the Heisenberg group and explain the relation between this group and the quaternionic analogous of the Siegel upper half space. We discuss regular functions, that are counterpart of complex holomorphic functions for quaternionic setting. The Hardy space is the space of regular functions in the Siegel upper half space with L^2-boundary values. We construct the Cauchy-Szegeo kernel for the Cauchy-Szegeo projection integral operator from the space

of L^2-integrable functions defined on the boundary of the quaternionic Siegel upper half space to the space of boundary values of the quaternionic regular functions of the Hardy space over the quaternionic Siegel upper half space. We also present the fundamental solution for a hypoelliptic operator related to the boundary of the Siegel upper half space.

Andrei Marshakov. **Differential equations and representations of almost Lie algebras.**
I am going to discuss some issues from representation theory of infinite-dimensional Lie algebras and *W*-algebras, which can be clarified in the context of recently found isomonodromy/CFT correspondence. In particular, their vertex operators just come from solutions of differential equations, and for the *W*-algebras of higher ranks this seems to be the only known general definition at the moment, fixed by corresponding monodromy data.
The talk is based on joint works with Pasha Gavrylenko and Misha Bershtein.

Vladimir S. Matveev. **Finsler metrics of constant curvature and integrable systems.**
I will mostly speak about Finsler metrics of positive constant flag curvature (I explain what is it) on closed 2-dimensional surfaces. The main result is that the geodesic flow of such a metric is conjugate to that of a Katok metric (that are well-understood examples of two-dimensional Finsler metrics of positive constant flag curvature). In particular, either all geodesics are closed, and at most two of them have length less than the generic one, or all geodesics but two are not closed; in the latter case there exists a Killing vector field.
Multidimensional generalizations will be given; in particular, in all dimensions the topological entropy vanishes and the geodesic flow is Liouville integrable. I will also show that in all dimensions a Zermelo transformation of every metric of positive constant flag curvature has all geodesics closed. The theory integrable systems, and in particular their orbital equivalence, plays an essential role in the proof.
The results are part of a joint work with R.Bryant, P.Foulon, S.Ivanov and W.Ziller.

Oleg Morozov. **Deformations of infinite-dimensional Lie algebras, exotic cohomology and integrable nonlinear partial differential equations.**
An important unsolved problem in the theory of integrable systems is to find conditions guaranteeing existence of a Lax representation for a given PDE. The use of the exotic cohomology of the symmetry algebras opens a way to formulate such conditions in internal terms of the PDEs under the study. The talk will discuss the recent developments of this approach. I will consider examples of infinite-dimensional Lie algebras with nontrivial second exotic cohomology groups and show that the Maurer-Cartan forms of the associated extensions of these Lie algebras generate Lax representations for integrable systems, both known and new ones.

Yury Neretin. **Determinantal systems of differential equations.**
We discuss following systems of partial differential equations on the set of pxq-matrices $Z=(z_{kl})$. We take a matrix composed of all partial derivatives $\partial/\partial z_{kl}$, and consider all minors of this matrix of a fixed size r. We consider functions annihilated by all such minors (for $p=q=r=2$ we get the John equation). We discuss problems, where such systems arise in a natural way and their properties: $GL(p+q)$-invariance and formulas for all holomorphic solutions in a 'matrix ball' (a set of complex matrices with norm $\leq R$).

Peter Olver. **Symmetry groupoids and weighted signatures of geometric objects.**
In this talk, I will refine the concept of the symmetry group of a geometric object through its symmetry groupoid, which incorporates both global and local symmetries in a common framework. The symmetry groupoid is related to the weighted differential invariant signature of a submanifold, that is introduced to capture its fine grain equivalence and symmetry properties. The groupoid/signature approach will be connected to recent developments in signature-based recognition and symmetry detection of objects in digital images, including jigsaw puzzle assembly.

Stanislav Opanasenko. **Group Analysis of general Burgers - Korteweg - de Vries equation.**

The complete group classification problem for the class of (1+1)-dimensional rth order variable-coefficient Burgers-KdV equations is solved for arbitrary values of r greater or equal than two. We find the equivalence groupoids of this. Showing that this class and certain subclasses are normalized, we reduce the complete group classification problem for the entire class to that for the selected maximally gauged subclass, and it is the latter problem that is solved efficiently using the algebraic method of group classification. Studying alternative gauges for equation coefficients with equivalence transformations allows us not only to justify the choice of the most appropriate gauge for the group classification but also to construct for the first time classes of differential equations with nontrivial generalized equivalence group with equivalence-transformation components corresponding to equation variables locally depending on nonconstant arbitrary elements.

Joint work with A.Bihlo and R.Popovich (https://arxiv.org/abs/1703.06932).

Olga Rossi. **The variational multiplier problem for PDEs.**

The variational multiplier problem was formulated and solved for the case of a system of two ordinary second order differential equations by Jesse Douglas in his celebrated paper in TAMS in 1941. Since that time, many efforts have been made to elaborate generalizations to systems of three and more SODEs, and a few attempts have been made to consider also PDEs. Contrary to the original Douglas' approach who used purely methods of classical analysis, it turned out that jet geometry appears to be the most efficient tool to tackle this hard problem. It this talk I will refer some recent results which include a solution of this problem for ODEs and new ideas concerning generalization to PDEs.

Ian Roulstone. **Monge-Ampere geometry and the Navier-Stokes equations.**

Monge Ampere structures arising in the incompressible Euler and Navier-Stokes equations in two and three dimensions will be described. A one-parameter family of metrics (with time as the parameter) is associated with such structures, and the evolution of these metrics is examined in the context of the invariants of the fluid flow.

This is a joint work with Bertrand Banos and Volodya Roubtsov.

Alexey Samokhin. **Nonlinear waves in layered media.**

The localized soliton-like wave in layered media may be described by different equations inside different layers (e.g. by KdV in absence of dissipation in one layer and by kdV-Burgers inside the layer with dissipation). The transition effects are studied and presented graphically.

Eivind Schneider. **Differential invariants of self-dual conformal structures.**

Let M be an oriented 4D manifold with a pseudo-Riemannian metric g of signature (4,0) or (2,2). We give a description of invariants of self-dual conformal structures $[g]$ with respect to the group of diffeomorphisms $Diff(M)$. This is done in two different ways.

First we consider all conformal metrics satisfying $*W=W$, where W is the Weyl tensor. Locally these are solutions to a system of 5 differential equations in 9 unknown functions, which is then factored by the pseudogroup $Diff_{loc}(M)$. The other method (applicable only in split-signature) uses a normal form of (anti-) self-dual metrics due to Dunajski, Ferapontov and Kruglikov, in which the self-duality equation is written as a system of 3 second order differential equations in 3 unknown functions and we factor this system by its symmetry pseudogroup.

Joint work with Boris Kruglikov.

Josef Silhan. **On projective metrizability in low dimensions.**

The problem of metrizability of an affine connection ∇ is to determine whether ∇ is a Levi-Civita connection of some metric(s). Projective structure can be given as a class ∇ of affine connections sharing the same system of (unparametrized) geodesics. In the case of projective metrizability one asks whether there is a metrizable connection in ∇. Optimally, the answer should be formulated in terms of projective curvature invariants. We shall present new results in dimensions three and four.

Arvid Siqveland. **Noncommutative Geometric Invariant Theory.**
Ordinary deformation theory for modules can be used to define formal moduli schemes in the ordinary commutative situation. Generalizing to noncommutative deformation theory, defining local formal moduli for families of modules, we can generalize to noncommutative affine schemes. We prove that in several cases, the noncommutative moduli schemes are noncommutative geometric quotients of group actions.

Jan Slovák. **Remarks on curvature in sub-Riemannian geometry.**
We shall deal with the sub-Riemannian geometries where the distribution defines a parabolic geometry and we shall discuss local invariants of the sub-Riemannian metrics. Technically, this leads to study of some special Lie algebra cohomology. In particular, we shall illustrate this approach on the free step-2 distributions.
Based on an ongoing work with Dmitri Alekseevsky and Sasha Medvedev.

Eldar Straume. **Shape as the basis for dynamics, epitomized by the planar three-body problem.**
For planar motions in the three-body problem, any small piece of the shape curve on the shape sphere (assuming the curvature is not constant), for a given value of the angular momentum, is sufficient to generate the Newtonian motion in the place (up to a congruence). In particular, the time parameter is generated from pure shape.

Dennis The. **Homogeneous integrable Legendrian contact structures in dimension five.**
Given a contact manifold, a splitting of the contact distribution into a direct sum of two Legendrian sub-distributions is called a Legendrian contact structure. When both sub-distributions are integrable, these structures are locally equivalently described as compatible, complete systems of 2^{nd} order PDE for a scalar function (considered up to point transformations), and so this setting is a natural generalisation of the classical study of scalar 2nd order ODE.
I will discuss recent work with Boris Doubrov and Sasha Medvedev in which we gave a complete classification of (complex) homogeneous such structures in dimension five having at least one-dimensional isotropy.

Maria Ulan. **What is common between parallel parking, falling cat and inverse Kapitza pendulum?**
I will present how based on prof. A.Vinogradov's approach one could describe different types of complex motion. In particular, I will discuss following situations:
1. Configuration space as a space with singularities: linkages, manifolds with corners.
2. Fixed motion of subsystem: inverse Kapitza pendulum as a nonholonomic system.
3. Reachability problems: parallel parking and falling cat.

Alexander Verbovetsky. **Toward a geometry of nonlocal Hamiltonian structures**
Recursion operators of PDEs can be understood as Bäcklund auto-transformations of the tangent coverings. In a similar vein, nonlocal Hamiltonian operators can be identified with Bäcklund transformations between tangent and cotangent coverings. In this talk I will discuss all this from the geometric viewpoint, and will speculate on the possible nonlocal Jacobi identity.
Joint work with Joseph Krasil'shchik.

Luca Vitagliano. **Holomorphic Jacobi Manifolds.**
After an Introduction to Jacobi manifolds and their role in Poisson Geometry, we introduce and study holomorphic Jacobi structures from a real differential geometric point of view. We also discuss the relationship with Generalized Geometry (in odd dimensions). Our analysis parallels a similar one for

holomorphic Poisson structures by Laurent-Gengoux, Stianon and Xu. However, the situation appears more involved in the Jacobi case.
This is a joint work with Assa Wade.

Andreas Vollmer. **Classification of metrics admitting one essential projective vector field.**
In 1882 Sophus Lie posed the problem of classifying 2-dimensional (pseudo-) Riemannian manifolds admitting one or several projective vector fields, i.e. vector fields whose local flow sends geodesics to geodesics (viewed as unparametrized curves). The case when the projective action is locally transitive has been completely understood (around regular points) by Bryant, Manno & Matveev in 2007, whereas in the non-transitive case only partial results have been known (cf. Matveev, 2012).
The talk presents joint research with G. Manno.

Henrik Winther: **Sub-maximally Symmetric Quaternionic and Quaternionic-Hermitian Structures.**
The symmetry dimension of an almost quaternionic structure on a manifold is the dimension of its full automorphism algebra. Let the quaternionic dimension n be fixed. The maximal possible symmetry dimension is realized by the quaternionic projective space $\mathbf{HP}^n$, which has symmetry group G=PGL($n+1$,**H**) of dimension $dim(G)=4(n+1)^2-1$. An almost quaternionic structure is called submaximally symmetric if it has maximal symmetry dimension amongst those with lesser symmetry dimension than the maximal case.
We show that for $n>1$, the submaximal symmetry dimension is $4n^2-4n+9$. This is realized both by a quaternionic structure (torsion free) and by an almost quaternionic structure with vanishing Weyl curvature. This is a joint work with Boris Kruglikov and Lenka Zalabová.
We also consider the quaternionic-Hermitian case, where there exists a compatible metric g, and the symmetry algebra is required to preserve this metric. We show that the sub-maximal model is unique, and its symmetry dimension is $2n^2+n+4$. This part is joint with Boris Kruglikov.

Martin Wolf. **Higher groupoid bundles and higher gauge theory.**
Recent developments in formulating higher gauge theory with higher groupoids as gauge structure will be reviewed. The approach makes use of simplicial geometry and develops higher gauge theory from first principles. As such it captures a wide range of theories including ordinary gauge theories and gauged sigma models as well as their categorifications. It will also be explained how these ideas can be combined with those of twistor theory to obtain non-Abelian self-dual tensor field theories in six dimensions and Yang-Mills theory in four dimensions.

Ori Yudilevich. **A Modern Approach to Cartan's Structure Theory for Lie Pseudogorups.**
In two pioneering papers dating back to 1904-05, Elie Cartan introduced a structure theory for Lie pseudogroups, generalizing Sophus Lie's structure theory for the special class of Lie pseudogroups of finite type. While the finite case has evolved into a mature and rigorous theory -- the theory of Lie groups – Cartan's general theory has not reached that same level of maturity. In this talk, I will present a modern formulation of the theory, encompassing Cartan's three fundamental theorems for Lie pseudogroups, the notion of prolongation and a reduction algorithm by the so-called "systatic system". Two of the key ingredients in this formulation are jet spaces and Lie groupoids/algebroids.
This talk is based on joint work with Marius Crainic.

Lenka Zalabova. **Generalized path geometries, robots and a bit of Maple.**
We introduce a plane mechanism such that its kinematic space forms a parabolic geometry known as a generalized path geometry in dimension 7. We study the nilpotent approximation of such a geometry. In particular, we get a nilpotent Lie group, which is a 2-step Carnot group, and we study corresponding sub-Riemannian control system.