2018 Winter School in Geilo, Norway: Geometry, Analysis, Physics

Mini-Courses:

Sub-Riemannian structures and analysis of induced operators

by Wolfram Bauer (Leibnitz University of Hannover)

A variety of problems in mathematics, physics or applied sciences can be reduced to the question of whether one can steer a system under non-holonomic constraints from a given initial state to any final state. In the case of existence, one may look for a connecting path in the configuration space of minimal length or energy. Imagine a cat falling from a roof with the back ahead and zero angular momentum. By changing her shape, the cat is able to rotate around 180 degrees and safely land on her feet. How is this possible and what is the most efficient movement? In the framework of sub-Riemannian geometry, problems of such type are formulated and discussed in a mathematically rigorous way. The questions arise whether a given manifold carries a sub-Riemannian geometry and how to classify such structures. From the viewpoint of analysis, sub-Riemannian structures often induce intrinsic sub-elliptic operators such as sub-Laplacians. These can be analyzed via functional analytic and PDE methods. Thus, one may study geometry through analysis or vice versa. A typical question is: "Which geometric data can be observed from the spectrum or the heat kernel of a sub-Laplacian?" Compared to the elliptic theory of the Beltrami-Laplace operator on a Riemannian manifold, a variety of new effects can be observed in both the geometric and analytic picture.

In the first part of this mini course, I am planning to provide a basic introduction to sub-Riemannian geometry. Various examples will be discussed. In the second part, we consider concrete geometric operators and their properties from an analytic point of view. The course should be accessible to students with a basic knowledge of functional analysis, manifold theory and PDE.

Special geometry and supersymmetry

by Vicente Cortés (University of Hamburg)

The scalar geometry of supersymmetric field theories with eight real supercharges is known as *special geometry*. I will describe this geometry and various physics-inspired constructions in the language of differential geometry without assuming any knowledge in physics. The main results include global structure theorems in special geometry and new explicit examples of complete quaternionic Kähler manifolds.

1) Introduction to special geometry
   (Physical motivation, affine and projective special real manifolds, affine and projective special Kähler manifolds, hyper-Kähler and quaternionic Kähler manifolds)

2) Geometric constructions relating different special geometries I
   (Affine and projective versions of the r- map, affine and projective versions of the c-map, some global properties)

3) Geometric constructions relating different special geometries II
(One-loop quantum correction, HK/QK-correspondence; special geometry of Euclidean supersymmetry)

4) Construction of complete quaternionic Kähler manifolds
(Characterization of complete projective special real manifolds, classification results; completeness criteria for projective special Kähler manifolds; completeness results for the one-loop corrected Ferrara-Sabharwal metric; examples of complete quaternionic Kähler manifolds of small co-homogeneity)

Geometrical Lie superalgebras and their applications

by Andrea Santi (Universita di Bologna)

L1 Motivating supersymmetry and supergravity.
L2 Supersymmetric supergravity backgrounds (in d=11)
L3 The Killing superalgebra and its algebraic structure
L4 Deformation theory of Lie superalgebras
L5 Generalised Spencer cohomology

Lie superalgebras of geometrical origin play an important rôle in certain areas of Physics. The purpose of these lectures is to introduce supersymmetry and supergravity, motivating the study of its supersymmetric backgrounds in the context of the unique supergravity theory in eleven dimensions. We will discuss the homogeneity conjecture (now essentially a theorem) which states that solutions preserving more than half of the supersymmetry are homogeneous. The key ingredient in the proof of this result is the construction of a Lie superalgebra generated from spinor fields satisfying a certain PDE. We will discuss the algebraic structure of this so-called Killing superalgebra and some algebraic techniques to construct such Killing superalgebras and, in the homogeneous case, to reconstruct the geometry. I will summarise recent work in collaboration with Andrea Santi and Paul de Medeiros and its applications to the classification of supersymmetric supergravity backgrounds and to the determination of geometries admitting rigidly supersymmetric field theories. I do not assume any knowledge of physics: all relevant definitions will be given along, I hope, with sufficient motivation.

Remez' inequality and estimates of solutions to second order elliptic equations.

by Eugenia Malinnikova (NTNU)

We will start with the classical Remez inequality for polynomials and see how it can be applied to propagation of smallness for real analytic functions and solutions of continuous and discrete elliptic equations. Then we will discuss various generalizations of the inequality. In particular, we obtain a version for solutions of general second order elliptic PDE, where the degree of a polynomial is replaced by the frequency of the solution.

Short communications:

(4, 7)-decomposable solutions of 11-dimensional supergravity

by Ioannis Chyrsikos (INdAM – Torino University)
(joint project with Dmitri Alekseevsky and Arman Taghavi-Chabert)

We examine the supergravity equations on 11-dimensional oriented Lorentzian manifolds given by a product of an oriented 4-dimensional Lorentzian manifold and an oriented 7-dimensional Riemannian manifold. For a specific type of flux forms, we show the Maxwell equation can be read in terms of special 3-forms on the Riemannian 7-manifold M^7 in particular the supergravity Einstein equation on M^7 reduces to the Einstein equation with an extra stress-energy tensor associated to such 3-forms. We also prove that for stable 3-forms the Maxwell equation can induce a weak G_2-structure and then obtain decomposable supergravity backgrounds given by the product of a weak G_2-manifold with a Lorentzian Einstein manifold 4-manifold. We finally discuss applications in the homogeneous case.

The teleparallel trick

by John Huerta (Instituto Superior Técnico)

General relativity describes gravity using the Levi-Civita connection: a torsion-free connection with typically nonvanishing curvature. Teleparallel gravity describes gravity using a flat connection with typically nonvanishing torsion. It is locally equivalent to general relativity. We describe this equivalence, and how to extend it to D=4 N=1 supergravity.

Black Hole Entropy and its Non-Linear Mysteries

by Alessio Marrani ("Enrico Fermi" Center, Roma, IT)

Freudenthal duality can be defined as an anti-involutive, non-linear map acting on symplectic spaces. After a general introduction on some aspects of extended (super)gravity theories in four dimensions and the structure of their U-orbits, I will consider their U-duality Lie groups "of type E7", and the corresponding notion of Freudenthal duality. I will elucidate and comment on the relation between the Hessian of the black hole entropy and the pseudo-Riemannian, rigid special Kaehler metric of the pre-homogeneous vector spaces associated to the U-orbits.

On a class of 3-algebras

by Andrea Santi (Universita di Bologna)

In this seminar I will talk about a class of 3-algebras (=algebras with a ternary product) originally introduced by I. L. Kantor in the 1970s as a natural generalization of Jordan triple systems. I will describe the classification problem of simple Kantor 3-algebras (over the complex field) and propose a solution to this problem. I will show that, under appropriate topological conditions, every simple Kantor 3-algebra is finite-dimensional and then give a classification in terms of real forms of complex simple Lie algebras. The Kantor 3-algebras of exceptional type can be divided into three main classes and a concrete example will be given for the most interesting class. This is a joint work with N. Cantarini and A. Ricciardo.