Master course
Topology, Homology, Lie

Fall Semester 2020
[Master code MAT-3810; PhD code MAT-8810]

7.09.
Lectures: L1 and L2. L3: L3.1-3.3.
Classical topological spaces and basic operations with them. Homotopy.
Exercises. 1.1-5, 1.19-20 (1.9-18 solved in class), 2.1, 2.4, 2.14-15, 2.18 (2.3, 2.7-10, 2.12, 2.17 solved in class).

14.09
Lectures: L5: 5.1-4 (no flag manifolds) + (only formulations): 5.5 (Theorem), 5.6 (Corollary 2), 5.7 (first theorem), 5.9 (first theorem). L6: 6.1-3, 6.4, 6.6, 6.8-9. L7: 7.6. L8: 8.1-3.
CW spaces (cell decomposition). Fundamental group. Coverings.
Exercises: 5.7, 5.13-14, 5.15-16 [More: construct Hasse diagrams of cellular decompositions of Gr(2,n) for any n including infinity], 5.18, 5.20. L6: 6.2, 6.4, 6.5-6. L7: 7.18.

21.09
Lectures: L5: 5.5 (first four lines), 5.7 (first two lines). L6: 6.5 (formulation of the Lifting theorem). L8: 8.4, 8.7-8.8. L9: 9.1-9.2, 9.4, 9.7 (formulation of the theorem), 9.8-9.9. L10: 10.1 (formulation of the Freudental theorem), 10.2-10.3.
Universal covering spaces and fundamental groups. Higher homotopy groups. Relative homotopy groups and long exact sequence of a pair. Fiber bundles and long exact sequence.
Exercises: 8.4, 9.10-11.

28.09
Lectures: L12: 12.1-12.5, L13: 13.1-13.2.
Singular homology. Cellular homology. Their equivalence.
Exercises: previously unsolved plus: prove Five-Lemma in L8.8 without using book-proof + 8.16; 12.5-6.

5.10
Lectures: L13: 13.1-13.6, 13.8, L15: 15.1-15.2.
Computations of cellular homology for classical spaces. Classical (oriented and non-oriented) surfaces (without boundary), their cell decompositions and computation of homology.
Exercises (plus from before): Five-Lemma, 12.5-6, 13.8, 13.11, 13.13

12.10
Lectures: 15.2-15.5.
Homology with coefficients and cohomology.
Exercises: No exercises due to part of the group trip to UiB.

Assignment-1:

- Compute the Schubert cell decomposition (via Young tableaux, Hasse diagram etc) for Gr(3,6)
- Consequently compute homology and cohomology of the complex Grassmanian Gr_C(3,6).
- Derive the Euler characteristic of Gr_C(3,6) in two (equivalent) ways.

19.10
Lectures: 15.6, 14.2-14.3, 14.5.
Homotopy equivalence. Relation of homology and homotopy. Gourevich and Poincare theorems. UCF: universal coefficients formula.
Exercises: compute homology of RP^n with coefficients in Z_m for any m (m=2, m odd, even m>2), compute homology and cohomology of classical surfaces, orientable or not

26.10
Lectures: L16: 16.1-16.3, 16.5.
Ring structure in cohomology via x-product and cup product in chain spaces and induced cohomology maps. Other multiplications: cap product, intersection form in homology.
Exercises: 14.1-4, 14.9, 14.11; Prove the formula for d(c1xc2), the formula for pairing of c1xc2 and a1xa2 and derive the formula for \partial(a1xa2); Using multiplication in cohomology prove that the bouquet of S2 and S4 is not homeomorphic (even not homotopy equivalent) to CP^2

2.11
Lectures: L17: 17.1-4.
Manifolds, pseudo-manifolds and Poincare duality. Nondegeneracy of the intersection form.
Exercises: Finish those from the previous session.

9.11
Lectures: Outlook from L20-L21 and other sources.
Spectral sequences (review) + Lie algebras and Lie groups (review).
Exercises: Compute homology and cohomology of real Gr(2,4) over Z, Z_2 and R. Relate them via UCF, compute the Euler characteristic and check the Poincare duality.

Assignment-2:

- Compute homology and cohomology of real Grassmanian X=Gr(3,6) over Z, Z_2, R.
- Relate H^* to H_* via UCF and use the latter to obtain H_*(X,Z_3).
- Compute the Euler characteristic of X in two ways and check the Poincare duality.
- Distinguish S^2\vee S^4 from CP^2 using the cohomology ring (group will not suffice).
- Derive that S^2\vee S^4 is not homotopy equivalent to a closed oriented manifold.

2.12
Exam: 1-3 candidates

Literature (main refs to the book [1]):
1. Anatoly Fomenko and Dmitry Fuchs “Homotopical Topology” (UiT online library)
2. Notes by G.Quick: http://folk.ntnu.no/gereonq/MA3403_Lecture_Notes.pdf
3. DG library for Maple.